STUDY OF THE TURBULENT FLOW OF A GAS OVER LONG BLUNT BODIES IN A VISCOUS SHOCK LAYER

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UDC 532.517.4:518.12

A number of problems connected with improving traditional configurations are being encountered at the current stage of supersonic aircraft design. Designers are searching for new three-dimensional shapes having optimum aerodynamic characteristics at hypersonic velocities. In connection with the attempt to develop hypersonic aircraft, it is interesting to study fluid flow over plane elements – wings – at high supersonic velocities within a broad range of Reynolds numbers. With an increase in velocity and pressure, an increasingly large role is played by aerodynamic heating and friction on the surface of the body as it encounters a turbulent flow regime in a shock layer. In connection with this, it is necessary to study the effect of the geometry of the body on the distribution of heat flux, friction, the path of the shock wave, and other flow parameters. In limiting cases, a three-dimensional body such as an elliptical paraboloid is transformed into an axisymmetric body such as a paraboloid of revolution (with equal ellipse semi-axes) or a parabolic cylinder (if one semi-axis of the ellipse approaches infinity).

In contrast to studies made earlier (see the survey [1], for example) which examined laminar flow regimes within the framework of the equations of a viscous shock layer, our goal here is to develop a single algorithm for calculating the parameters of viscous flow over long plane or axisymmetric bodies for a broad range of flow regimes – from laminar to turbulent. In the literature [1], most of the attention has been given to study of flow and heat transfer in the neighborhood of the forward critical point of axisymmetric bodies. In the case of prolate bodies, calculation of the integral drag and heat-transfer coefficients characterizing their motion and heating in the atmosphere requires determination of the gas-dynamic parameters along the entire generatrix of the body to its midsection.

We will numerically examine hypersonic flow over a long blunt plane body (a parabolic cylinder) or an axisymmetric body (paraboloid of revolution) within the framework of the model of a viscous shock layer in a turbulent flow regime. This model is known [1] to be suitable for calculation of flow over bodies for which there is no Newtonian separation point and no surface curvature discontinuity. Paraboloids and parabolic cylinders meet these requirements, so they will be used as examples in our calculations. The parameters of flow about other bodies such as blunt cones (or wedges) may in some cases be comparable to the parameters for flow over a specially chosen paraboloid (or parabolic cylinder). It has been suggested that such a correspondence exists for cones and wedges with different flare half-angles.

1. We will examine the hypersonic flow of a viscous gas over long smooth blunt plane bodies and solids of revolution. The system of equations describing the turbulent hypersonic viscous shock layer are obtained from the averaged Navier–Stokes equations. When written in dimensionless variables in a system of orthogonal curvilinear coordinates x, y connected with the surface of the body, the equations have the form [1]

$$\frac{\partial}{\partial x}(\rho r_w^\nu u) + \frac{\partial}{\partial y}(\rho r_w^\nu v) = 0,$$

$$\rho Du = -\varepsilon\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\frac{\mu_\Sigma}{K}\frac{\partial u}{\partial y}\right),\quad \rho\varkappa u^2 = \frac{\partial p}{\partial y},$$

$$\rho DT = 2\varepsilon\frac{\partial p}{\partial x} + \frac{2\mu_\Sigma}{K}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\mu_\Sigma}{\sigma_\Sigma K}\frac{\partial u}{\partial y}\right).$$

Moscow. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 69-75, January-February, 1993. Original article submitted June 19, 1990: revision submitted October 9, 1991.

 0021-8944/93/3401-0066$12.50

$$p=\rho T,\ D=u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y},\ K=\varepsilon\,\mathrm{Re},\ \varepsilon=\frac{\gamma-1}{2\gamma},\ \mathrm{Re}=\frac{\rho_\infty V_\infty R_0}{\mu_0},\ \gamma=\frac{C_p}{C_V},\ \sigma_\Sigma=\frac{\mu_0\mu_\Sigma C_p}{\lambda_\Sigma},\ \mu=\mu(T_0)=T_0^\omega,\ T_0=\frac{V_\infty^2}{2C_p},\ (1.1$$

where ν = 0, 1 for the plane and axisymmetric cases, respectively; $V_\infty u$, $\varepsilon V_\infty v$ are components of the velocity vector corresponding to the x and y axes; $\rho_\infty V_\infty^2 p$, $\varepsilon^{-1}\rho_\infty\rho$, T_0T, $\mu_0\mu_\Sigma$, λ_Σ are the pressure, density, temperature, viscosity coefficient, and thermal conductivity of the gas; and κ is the longitudinal curvature of the surface of the body. All of the linear dimensions are referred to the characteristic linear dimension R_0, while the normal coordinate is referred to εR_0. As R_0, we chose the radius of curvature of the blunt body at x = 0. The subscript w pertains to quantities on the surface of the body. The letter Σ represents the overall transport coefficients, accounting for molecular and turbulent transport.

The equations of the turbulent viscous shock layer (1.1) must be supplemented by boundary conditions on the shock wave and the surface of the body. In the case of uniform flow over the body, the modified Rankine–Hugoniot equations for the shock wave can be written as [2]

$$\begin{gathered}y=y_s(x)\colon\ \rho\left(v-u\frac{dys}{dx}\right)=v_\infty,\ p=v_\infty^2,\ v_\infty(u-u_\infty)=\frac{\mu_\Sigma}{K}\frac{\partial u}{\partial y},\\ v_\infty(T+u^2-1)=\frac{\mu_\Sigma}{\sigma_\Sigma K}\frac{\partial T}{\partial y}+\frac{2\mu_\Sigma}{K}u\frac{\partial u}{\partial y},\ u_\infty=\cos\alpha,\ v_\infty=-\sin\alpha\end{gathered}\qquad(1.2)$$

(α is the angle between a tangent to the surfce of the body and the symmetry axis). The following contact conditions are assigned on the impermeable surface of the body:

$$y=0\colon\ u=0,\ v=0,\ T=T_w(x).\qquad(1.3)$$

System (1.1-1.3) is unclosed, since we do not know the turbulent transport coefficients. Closure of the system requires making assumptions that would allow us to establish the relationship between these quantities and the mean values. Here, we will use the algebraic model in [3]. In accordance with the latter, the overall viscosity coefficient and thermal conductivity are linear combinations of the molecular and molar (turbulent) transport coefficients: $\mu_\Sigma = k_1\mu + k_2\mu_T$, $\lambda_\Sigma = k_1\lambda + k_2\lambda_T$. The coefficients k_1 and k_2 were chosen so that $\mu_\Sigma \to \mu$ in the laminar region and $\mu_\Sigma \to \mu_T$ in the turbulent region. The coefficients k_1 and k_2 are given by the formulas

$$k_1=\frac{1}{1+k^2(\eta_1/\eta_k)^2},\quad k_2=\frac{k^2(\eta_1/\eta_k)^2}{1+k^2(\eta_1/\eta_k)^2}$$

(k = 0.4 is the Karman constant).

Eddy viscosity is given by the Prandtl formula

$$\mu_\mathrm{T}=\rho l^2\left|\frac{\partial u}{\partial y}\right|,$$

where ℓ (the mixing length) is calculated from the formula [4]

$$\frac{l}{\delta}=0{,}1\,\frac{1-\exp(-8y/\delta)}{1+\exp(-6y/\delta)}$$

(δ is the thickness of the boundary layer).

The coefficients of total viscosity have the form

$$\begin{gathered}\mu_\Sigma=\mu\frac{\eta_k^2+k^2\eta_1^2r_t}{\eta_k^2+k^2\eta_1^2}=\mu\varphi(\eta_k,r_t),\\ \varphi(\eta_k,r_t)=\frac{(r_t^2-\eta_k^2)+\sqrt{(r_t^2-\eta_k^2)^2+4\eta_k^2r_t}}{2r_t^2},\\ r_t=\frac{\mu_\mathrm{T}}{\mu}=\frac{\rho l^2}{\mu}\left|\frac{\partial u}{\partial y}\right|,\ k^2\eta_1^2=r_t\varphi(\eta_k,r_t).\end{gathered}\qquad(1.4)$$

Total thermal conductivity can be represented as

$$\lambda_\Sigma = C_p\left(\frac{k_1}{\sigma}\mu + \frac{k_2}{\sigma_T}\mu_T\right), \tag{1.5}$$

if we introduce the laminar Prandtl number $\sigma = \mu C_p/\lambda$ and turbulent Prandtl number $\sigma_T = \mu_T C_p/\lambda_T$.

The expression for total viscosity [4] includes the parameter η_k, characterizing the thickness of the viscous sublayer. The choice made for this parameter to a large extent determines the results of the calculations. In the present study, we determine η_k by using a parabolic approximation of the Reynolds number Re_θ constructed from the momentum thickness [5]:

$$\begin{aligned}
&\eta_k = \eta_0 + c(\lg Re_\theta - 4) + b(\lg Re_\theta - a)^2,\\
&a = 3{,}2,\ b = 57,\ c = 3,\ \eta_0 = 10,\\
&b = 0\ (\lg Re_\theta > a),\ c = 0\ (\lg Re_\theta > 4).
\end{aligned}$$

An analysis of this formula shows that it refines the results only for low Re_θ.

To numerically solve the given problem, we write system (1.1-1.3) in Dorodnitsyn variables:

$$\begin{gathered}
\xi = x,\ \eta = \frac{1}{\Delta}\int_0^y \rho dy,\ \Delta = \int_0^{y_s} \rho dy,\\
u = u_0(\xi)\frac{\partial f}{\partial \eta},\ T = T_0(\xi)\frac{\partial \theta}{\partial \eta},\ \rho r_w^\nu v = -\frac{\partial \psi}{\partial y},\ \psi = r_w^\nu \Delta u_0 f.
\end{gathered} \tag{1.6}$$

Here, ψ is the stream function; and u_0 and T_0 are assumed to be equal to u_∞ and v_∞^2, respectively, since in the case of high Reynolds numbers ($K \to \infty$) it follows from boundary conditions (1.2) at $y = y_s$ that $u \to u_\infty$, $T \to v_\infty^2$. Also, singularities which arise in the momentum equation at the critical point $\xi = 0$ are solved at $u_0(\xi) = u_\infty$.

We numerically integrated the resulting boundary-value problem by a method [2] based on an implicit finite-difference scheme [6] with a high order of accuracy. The difference equations were solved by the trial-run method with iterations at each ξ step. The difference equations were linearized using the values of the functions obtained in the previous iteration.

The procedure used in numerically integrating the system in each iteration was as follows. We assinged a linear initial approximation for $f(\eta)$ and $\theta(\eta)$ on the critical line. We then determined the profiles of pressure p and the pressure gradient p_2. The pressure gradient $p_2 = \frac{1}{u_0}\frac{\partial p}{\partial \xi}$ was found from an equation obtained by differentiation of the momentum equation in a projection on a normal with respect to the parameter η. In Dorodnitsyn variables (1.6) for p and p_2, we obtained first-order ordinary differential equations which we integrated from the shock wave to the surface of the body by means of Simpson's formula. Density ρ was determined from the equation of state. Total viscosity μ_Σ was found from Eqs. (1.4), molecular viscosity μ was determined from the Sutherland conservation law, and total thermal conductivity was found from (1.5). We then integrated the equations for the corrected stream function f and temperature θ and calculated the new value for the decay of the shock wave in Dorodnitsyn variables Δ. The first boundary condition of system (1.2) was used to calculate Δ. Iterations on the given ray were performed until the maximum difference of all the profiles and Δ in a given iteration from the same in previous iteration was less than the specified accuracy.

2. Numerical calculations were performed in the transverse direction on a nonuniform grid which was made denser approaching the surface of the body. The total number of intervals was 59.

After solving the difference equations, we calculated the distributions of heat flux q_w and the friction coefficient C_f (heat flux was referred to $\rho_\infty V_\infty{}^3$, while friction was referred to $\rho_\infty V_\infty{}^2$) by means of the formulas

$$q_w = \sqrt{Re}\,\frac{\mu_\Sigma}{2\sigma_\Sigma K}\frac{\partial T}{\partial y}(y = 0),\ C_f = \sqrt{Re}\,\frac{\mu_\Sigma}{K}\frac{\partial u}{\partial y}(y = 0).$$

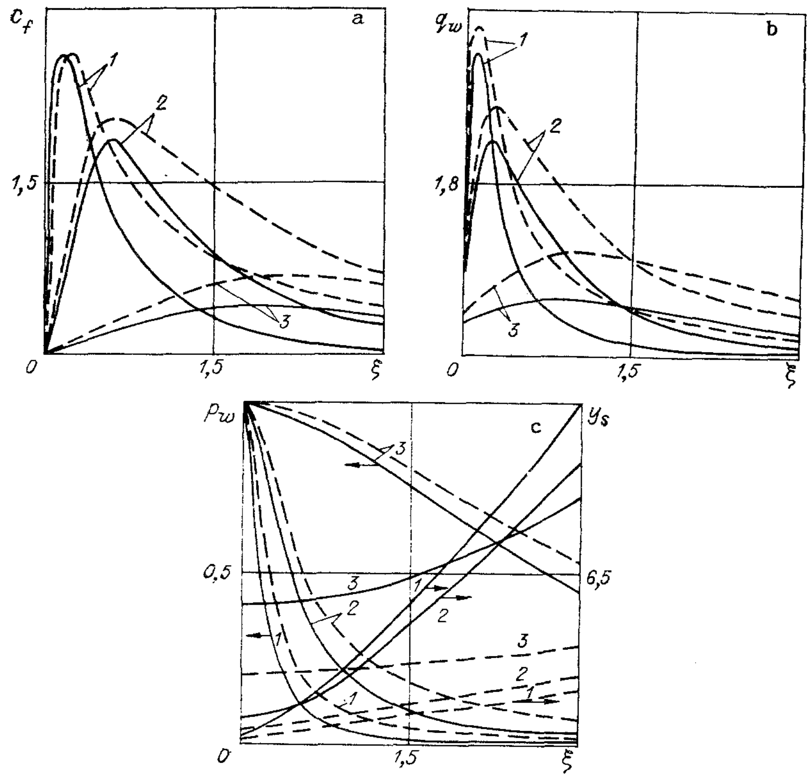

Fig. 1

In the analysis of the trajectory of the body and its total aerodynamic heating, an important role is played by integral characteristics: the overall coefficient of convective heat transfer

$$C_H = \frac{1}{S_m} \int_S \bar{q}_w dS \quad \text{for the solid of revolution,}$$

$$C_H = \frac{1}{r_m} \int_0^L \bar{q}_w dl \quad \text{for the plane body,}$$

the coefficient of wave resistance

$$C_D = \frac{1}{S_m} \int_S \bar{p}_w \sin \alpha \, dS \quad \text{for the solid of revolution,}$$

$$C_D = \frac{1}{r_m} \int_0^L \bar{p}_w \sin \alpha \, dl \quad \text{for the plane body,}$$

and the drag coefficient

$$C_\tau = \frac{1}{S_m} \int_S \bar{\tau}_w \cos \alpha \, dS \quad \text{for the solid of revolution,}$$

$$C_\tau = \frac{1}{r_m} \int_0^L \bar{\tau}_w \cos \alpha \, dl \quad \text{for the plane body,}$$

where S_m and S are the areas of the midsection of the solid of revolution and its lateral surface; L is the arc length; $r^2 = 2\beta z$; $r_m = \sqrt{2\beta z_L}$; z and r are the coordinates along the symmetry axis of the body and the normal to it; and β is a parameter.

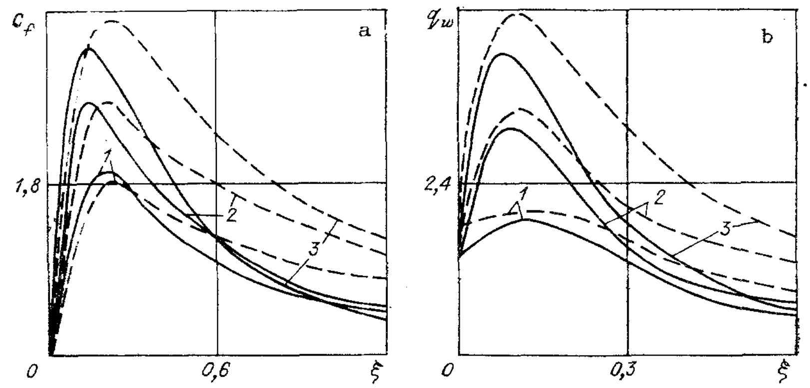

Fig. 2

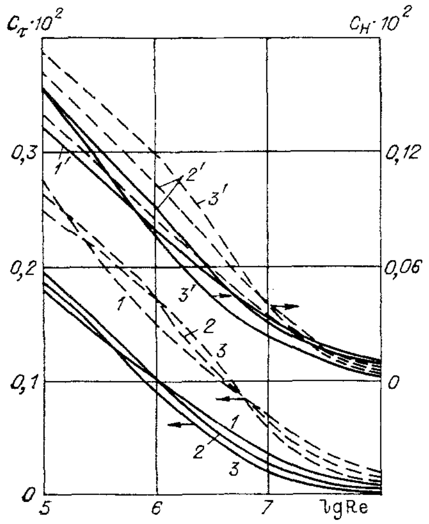

Fig. 3

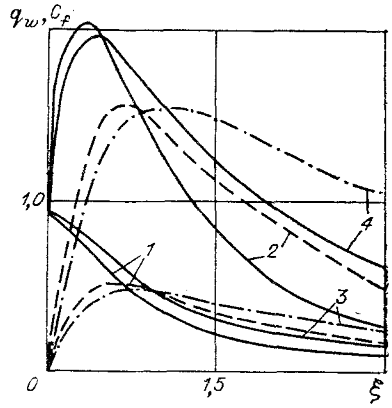

Fig. 4

$$\bar{q}_w = \frac{2\lambda_\Sigma \partial T/\partial y}{\rho_\infty V_\infty^3};\ \bar{p}_w = \frac{2p_w}{\rho_\infty V_\infty^2};\ \bar{\tau}_w = \frac{2\mu_\Sigma \partial u/\partial y}{\rho_\infty V_\infty^2}.$$

We assigned a body length $z_L = 15$ for all of the variants. Here, the length of the arc L is different for different values of β. By changing the parameter β, we can change the effective flare of the paraboloid and the parabolic cylinder. For greater convenience in interpreting the results of the calculations, the paraboloid and parabolic cylinder were made to correspond to a spherically blunt cone (or wedge) having the same blunting radius R_0 and midsection radius r_m. With an assigned length L, the flare half-angle β_c of such a cone (wedge) is found from the formula

$$\sin\beta_c = \frac{-(1-a)\,a + \sqrt{2a}}{1+a^2},\quad a = \frac{\beta}{z_L}.$$

Similarly, having assigned the half-angle β_c, we can unambiguously determine β:

$$\beta = z_L(1-\cos\beta_c)/(1-\sin\beta_c).$$

In the calculations we performed, we assigned both the parameter β and the half-angle of the equivalent cone (wedge) β_c. A numerical solution was obtained in the following ranges of the determining parameters of the problem: $10^5 \le \mathrm{Re} \le 10^8$, $\gamma = 1.4$, $T_w = 0.15$, $\omega = 0.5$, $0 < \beta < 5$, $\sigma = 0.7$, $\sigma_T = 0.75$.

Figures 1a-c and 2a, b show the distribution of the friction coefficient C_f, heat flux q_w, pressure p_w, and decay of the shock wave y_s over the surface of a parabolic cylinder (solid curves) and paraboloid (dashed curves). The results in Fig. 1 were obtained with $Re = 3 \cdot 10^6$ (lines 1-3 correspond to $\beta = 0.276$, ($\beta_c = 10°$), 0.69 (15°), 4.02 (30°), while the results in Fig. 2 were obtained with $\beta = 0.276$ ($\beta_c = 10°$) and $Re = 10^6$, $3 \cdot 10^6$, 10^7 (lines 1-3).

Figure 3 shows the coefficient of integral heat flux C_H and the total drag coefficient C_τ of the parabolic cylinder (solid curves) and paraboloid (dashed curves) in relation to Re for different β: curves 1, 1'-3, 3' correspond to $\beta = 0.276$; 0.69, and 1.375 ($\beta_c = 20°$). It is evident that the coefficients C_H and C_τ decrease monotonically with an increase in Re.

Figure 4 shows the distributions of q_w (solid curves) and C_f (dashed curves) on the surface of the paraboloid with $\beta = 1$ and $Re = 3 \cdot 10^6$ in the laminar and turbulent (lines 1, 2) regimes. Also shown are the distributions of $q_{w\ell}$ and $C_{f\ell}$ (solid curves 3, 4) and $q_{w\Sigma} = q_{w\ell} + q_{wt}$, $C_{f\Sigma} = C_{f\ell} + C_{ft}$ (dot-dash curves 3, 4). The heat fluxes $q_{w\ell}$, q_{wt} and friction coefficients $C_{f\ell}$, C_{ft} were calculated from analytical formulas in [2] for both laminar and turbulent regimes of flow in the boundary layer.

The numerical calculations showed that the form of the body, the flow regime (Reynolds number), and effective flare of the body have a significant effect on the dynamic and thermal characteristics of plane and axisymmetric bodies.

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